

# Divisibility

**Overview:** Do you know that 8 can be divided by 2 completely? The answer may be yes because it is a small number; therefore division can be carried out easily. However, can you figure out if 9,054,137,828 can be divided by 7 completely without a remainder?

Conducting long division for large numbers can be tedious (and boring). This lesson gives us an opportunity to answer such questions within seconds.

Divisibility is a concept which implies that a number should be divided by another completely. Since 8 can be divided completely by 2, we say it is divisible by 2. In this lesson, we will discuss quick tests that can make us say whether a number is divisible by 2, 3, 4, 5 and 7.

## Materials

- Pencil
- Paper

## Activity

### Divisibility test for 2

Let's start with an easy divisibility test: testing for the divisibility of 2. A number is divisible by two if the last digit is divisible by 2. Numbers that are divisible by two are 0, 2, 4, 6, and 8. If the last digit of your number is one of these, then the entire number is divisible by 2. Examples are 36, 92, 50, 18, and 46.

Circle the numbers that are divisible by 2: 24 90 67 76 6 49 26 32 313 358 927 1024 512

### Divisibility test for 3

This is an interesting test, because it seems mysterious how it works. To find out if a number is divisible by 3, we first *add* the digits of the number together and if the sum is divisible by 3, then the original number itself is also divisible by 3.

Let's try an example: 144

First, we'll figure out the sum of the digits:  $1 + 4 + 4 = 9$ .

Does 3 go into 9? You don't have to figure out how many times it does – it's just a yes or no thing.

Since the answer is *yes*, then 144 is divisible by 3!

Try this one before turning the page: Is 273,645 divisible by 3?

First, you need to sum the digits:  $2 + 7 + 3 + 6 + 4 + 5 = 27$

Does 3 go into 27? Yes!

So you've just figured out that 273,645 is divisible by 3.

#### Divisibility test for 4

A number is divisible by 4 if the last two digits are divisible by 4 (or if they are two zeros). For example, 2,016 is divisible by 4 since 16 is divisible by 4.

Is 244 divisible by 4?

The last two digits (244) form the number 44. When we look at the 44, we immediately know that 4 goes into 44, so that means that 244 is divisible by 4.

Is 18,336 divisible by 4?

Check out the last two digits. They form the number 36, and since 36 is divisible by 4 (again, it doesn't matter right now how many times 4 goes into 36), the number 18,336 is divisible by 4.

What about a million? Is 1,000,000 divisible by 4?

Yes, because the last two digits are "00" so automatically the number is divisible by 4.

#### Divisibility test for 5

This is probably the easiest of all divisibility tests. Simply look at the last digit on the number. If you find a 0 or 5 in the ones place, then the number is divisible by 5. Can you easily see that these numbers: 25, 24,510, 6,535 and 2,382,639,403,482,510 are divisible by 5?

What about these? 522,334, 524,251 and 63,734,638? (These are not divisible by 5.)

#### Divisibility test for 6

This one looks difficult, but it's really a combination of two previous tests we've just learned.

Remember that  $6 = 3 \times 2$ , which means that to get to 6, we must first multiply 3 and 2 together.

What this really means is that a number is divisible by 6 if it is divisible by *both* 2 and 3.

For example, is the number 1,134 divisible by 6? Let's find out.

First, we can test for the divisibility of 2. Since the last digit is a 4, this number is divisible by 2. So far, so good. But to be divisible by 6, it's also got to pass one more test.

Now let's test for divisibility by 3. We'll sum the digits  $1 + 1 + 3 + 4 = 9$ . Since 9 is divisible by 3, this number passed the second test.

The result? 1134 can be divided by 3 *and* 2, so it's also divisible by 6.

### Divisibility test for 7

There are a couple of ways to check for divisibility by 7. This one happens to be one of my favorites, and it makes use of a strange 3-2-1 pattern. Let's do an example so you see how it works:

Our job is to figure out if the number 6, 124 314 is divisible by 7.

Since the number has 7 digits, we write a pattern of 3, 2 and 1 beginning with 3 as shown in the left column of numbers. Next assign each number with a + or - sign, beginning with positive downward but shifting from positive to negative or negative to positive each time you hit a 1.

Now write the multiply symbol (x) after the 3-2-1 pattern of numbers, and then write your original number *backwards* so that the ones place gets multiplied by the +3, the tens place digit gets multiplied by the +2, and so forth. See if you can figure out what I've done here:

+3	+3 x 4	+3 x 4 = 12
+2	+2 x 1	+2 x 1 = 2
-1	-1 x 3	-1 x 3 = -3
-3	-3 x 4	-3 x 4 = -12
-2	-2 x 2	-2 x 2 = -4
+1	+1 x 1	+1 x 1 = -1
+3	+3 x 6	+3 x 6 = 18

Now add up all the numbers together in the last column to get 14 ( $12 + 2 + (-3) + (-12) + (-4) + (-1) + 18 = 14$ ). Since the number 14 is divisible by 7, then the number 6,124,314 is divisible by 7.

This is a tricky divisibility test, so let's try another example: Is 18, 102 divisible by 7? Try it in the space below before turning the page for the steps.

The number 18,102 has 5 digits, which means we will have a 3-2-1 pattern made up of 5 digits.

$$+3 \times 2 = 6$$

$$+2 \times 0 = 0$$

$$-1 \times 1 = -1$$

$$-3 \times 8 = -24$$

$$-2 \times 1 = -2$$

Sum up the numbers on the right side gives -21. Disregard the + or - sign and notice that 21 is divisible by 7. So the number 18,102 is divisible by 7!

What about other numbers? You can tell if a number is divisible by 9 if it's divisible by the numbers that make up 9 ( $9 = 3 \times 3$ ). So 270 is divisible by 9 since it's also divisible by 3. The number 100 is divisible by 10 ( $10 = 2 \times 5$ ) since it's also divisible by 2 and 5.

Now it's your turn! Work out the exercises below. (You'll find answers at the back of this book.)

### Exercises

1. Identify the number(s) that are divisible by 2 in the following list.  
301, 3645, 3673
2. Identify the numbers that are divisible by 3 in the following list.  
3981, 430, 4598, 72624
3. Which one of the following numbers is divisible by 7  
5894, 56723, 17259
4. Is 2353740 divisible by 4?
5. Which one of the following numbers is divisible by both 2 and 5?  
10002, 453970, 637385
6. A number is divisible of 2 and 3, will it be divisible by 5?
7. If a number is divisible by 2 only, will it be divisible by 4?
8. Which of the following numbers is divisible by 3?  
45769, 25784, 2391
9. List any three 3-digit numbers that are divisible by 5.
10. List two 4-digit numbers that are divisible by 4.

## Answers to Exercises: Divisibility

1. None
2. 3981,72624
3. 5894
4. Yes
5. 453970
6. It may or it may not
7. no
8. 2391
9. The numbers must be composed of 3 digits and their last digit must be 0 or 5
10. The numbers must be composed of 4 digits and their last two digit must be 00 or must form a number that is divisible by 4